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METHODS OF SOLUTION OF TRANSPORT EQUATION IN INHOMOGENEOUS AND FINITE MEDIA.

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At present Monte Carlo method is a basic one for the solution of the radiation transport problems in inhomogeneous (finite and infinite) media under conditions of energy dependence of the cross sections and in the process of anizotropic scattering. The main advantage of Monte Carlo method in comparison with the generally used numerical methods consists in suitability to multidimensional problems and brightly reveals in the transport theory problems.

In fact, the solution of a general kinetic equation for the flux $\phi(\vec{r},\vec{\Omega},E)$ depended upon six variables appears as insurmountable task. In many cases when it is required the calculation of the functionals of a certain kind the problem of multidimensionality is not facilitated by the theory of adjoint functions since the dimension of the problem is still too high.

At the same time according to the character of the information necessary to receive Monte Carlo method may be easily modified for the purpose of automatic elimination of a number of variables. Two variants of the method are developed in a given report. One of them deals with the problem of calculation of the flux $\phi(\vec{r},\vec{\Omega},E)$ for a certain restricted set of coordinates, directions and energies, and the other is applicable to the problem of determination of the functionals of a certain kind.

In many cases where radiation fields considered possess the high degree of symmetry it is reasonable to use the dif-

ference methods. Thus, for example, the numerical solution of the problem for the angular and spectral distributions of radiation at different distances from a point isotropic source is more effective in comparison with the well-known moments method. The region of application of difference methods may be significantly expanded if to use the theory of adjoint equations. The latter is valid in that cases where the functionals sought for depend on a lesser number of independent variables in comparison with the function describing space-energy and angular distributions of radiation.

The difference methods.

Plane-parallel problems. The solution of many problems on gamma-ray penetration through the material is reduced to determination of the function $N(z,\mu,\lambda)$ satisfying the equation 1.

$$\mu\frac{\partial N}{\partial z}+\Sigma N=\mathcal{F}+f, \tag{1}$$

where

$$\mathcal{F}=\int_0^{2\pi}d\alpha\int_{-1}^{+1}G(\xi,\lambda_1)N(z,\mu',\lambda_1)\,d\mu',$$

$$\xi=\mu\mu'+\sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos\alpha,\quad \lambda_1=\lambda-1+\xi \tag{2}$$

under supplementary conditions

$$\begin{cases} N(0,\mu,\lambda)=x(\mu\lambda), & \mu>0 \\ N(a,\mu,\lambda)=0, & \mu<0 \end{cases} \tag{3}$$

The functions f, x, Σ and G are supposed known. For the approximate solution of equation (1) let us divide interval $[0\le z\le a]$ into n parts, interval $[-1\le\mu\le 1]$ into m parts and along axis λ plott the points $\lambda_0,\lambda_1,\lambda_2,\dots,\lambda_p,\dots\lambda_o$ is a least wave lenght of primary gamma-rays. The points of division of the intervals $[0\le z\le a]$ and $[-1\le\mu\le 1]$ denote as z_k, $k=0,1,\dots,n$ and μ_ν, $\nu=0,1,\dots,m$, $z_0=0$, $z_n=a$, $\mu_0=1$, $\mu_m=-1$, respectively.

Approximate values of the function $N(z,\mu,\lambda)$ in the point (z_k,μ_ν,λ_p) denote as $N^p_{\nu k}$.

Similar notation is used for the other functions.

Let us return to the equation (1) and assume $\mu=\mu_\nu$ and $\lambda=\lambda_p$. Then we have

$$\mu_\nu\frac{dN^p_\nu}{dz}+\sum^p N^p_\nu=\mathcal{F}^p_\nu+f^p_\nu \tag{4}$$

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Explanation is not required for notation. The function $\mathcal{F}_\nu^\rho$ is defined by the integral (2) at $\mu = \mu_\nu$ and $\lambda = \lambda_\rho$.

Suppose the right side of equation (4) is known, then the problem is reduced to the integration of ordinary first order differential equation. Integrating this equation by z within (z_k, z_{k+1}) and using the quadrature trapezoid rule receive approximately *.

$$\mu_\nu (\mathcal{N}_{\nu k+1}^\rho - \mathcal{N}_{\nu k}^\rho) + \frac{\Sigma^\rho \Delta z_k}{2} (\mathcal{N}_{\nu k+1}^\rho + \mathcal{N}_{\nu k}^\rho) = \frac{\Delta z_k}{2} (\mathcal{F}_{\nu k+1}^\rho + \mathcal{F}_{\nu k}^\rho + f_{\nu k+1}^\rho + f_{\nu k}^\rho). \tag{5}$$

$\Delta z_k = z_{k+1} - z_k$, $k = 0, 1, \ldots, n-1$. Additional conditions following from the boundary ones (3) are necessary to add to the latter system

$$\begin{cases} \mathcal{N}_{\nu 0}^\rho = \dot{x}(\mu_\nu, \lambda_\rho), & \mu_\nu > 0, \\ \mathcal{N}_{\nu n}^\rho = 0, & \mu_\nu < 0. \end{cases} \tag{6}$$

After this, the system (5) is solved in the direction of increasing k, if $\mu_\nu > 0$, and in opposite direction, if $\mu_\nu < 0$ **.

Proceeding to the approximate calculation of the function $\mathcal{F}(z, \mu, \lambda)$ at the right of equation (4) note that its expression (2) representing usual record of the scattering integral is not suitable for our purposes. The method proposed is based on the other representation of this integral obtained by the substitution of integration variables and has the following form

$$\mathcal{F}(z, \mu, \lambda) = \int_0^{2\pi} d\alpha \int_{-1}^{+1} G(\xi, \lambda) \mathcal{N}(z, \mu, \lambda_1) d\xi, \tag{7}$$

where

$$\mu' = \mu\xi + \sqrt{1-\mu^2}\sqrt{1-\xi^2}\cos\alpha \tag{8}$$

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* It is supposed that Σ is not depend on z. This restriction is not substantial.

** The division of the interval $[-1 \le \mu \le 1]$ is performed in that manner that none of the points μ_ν is not fall on the point $\mu = 0$.

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Let $\mu = \mu_0$ and $\lambda = \lambda_p$ obtain

$$\mathcal{F}_\nu^p = \int_0^{2\pi} d\alpha \int_{-1}^{+1} G(\xi, \lambda_p - 1 + \xi) \mathcal{N}(z, \mu', \lambda_p - 1 + \xi) d\xi \qquad (9)$$

where μ' is calculated by formula (8) at $\mu = \mu_\nu$.

For calculation of the inner integral it may be constructed the quadrature formula with the points ξ_{pq} defined from the conditions

$$\lambda_p - 1 + \xi_{pq} = \lambda_q; \quad q = p-s, \quad p-(s-1), \ldots, p. \qquad (10)$$

Then, write down the integral (9) approximately in

$$\mathcal{F}_\nu^p = \pi A_{pp} \mathcal{N}_\nu^p + \sum_{q=p-s}^{p-1} A_{pq} \int_0^\pi \mathcal{N}(z, \mu_{pq}^\nu, \lambda_{p-q}) d\alpha. \qquad (11)$$

$$\mu_{pq}^\nu = \mu_\nu \xi_{pq} + \sqrt{1-\mu_\nu^2}\sqrt{1-\xi_{pq}^2}\cos\alpha \qquad (12)$$

Now, suppose that the values of the function $N(z, \mu, \lambda)$ at $\lambda = \lambda_q < \lambda_p$ are already found. Then, by substituting the relation (11) into the equation (4) and transfering $\pi A_{pp} \mathcal{N}_\nu^p$ into left obtain the following expression:

$$\mu \frac{d\mathcal{N}_\nu^p}{dz} + (\Sigma^p - \pi A_{pp}) \mathcal{N}_\nu^p = \sum_{q=1}^{m_p} A_{pq} \int_0^\pi \mathcal{N}(z, \mu_{pq}^\nu, \lambda_{p-q}) d\alpha \qquad (13)$$

The right side of the latter equation contains known functions and the problem is solved by the method decribed above. It remains to find the following integrals.

$$J_{pq}^\nu = \int_0^\pi \mathcal{N}(z, \mu, \lambda_{p-q}) d\alpha \qquad (14)$$

at $\mu = \mu_{pq}$. (See formula 12). For this purpose interval $[-1 \le \mu \le +1]$ is divided into two parts $(-1,0)$ and $(0,+1)$ and on each let be approximately

$$\mathcal{N}(z, \mu, \lambda_p) = \begin{cases} \sum_{s=0}^{z_1} a_{sp}(z) \mu^s, & \mu > 0, \\ \sum_{s=0}^{z_2} b_{sp}(z) \mu^s, & \mu < 0. \end{cases} \qquad (15)$$

Here, z_1 and z_2 are some constants and the coefficients a_{sp} and b_{sp} may be determined by different ways, for example, by the least-square method.

By substituting the expression (15) into (14) obtain the integrals which are not difficult to calculate. Thus, the determination of space and angular gamma-ray distributions for every new value of energy variable is reduced to solution of the ordinary differential equation (13) with known right side defined at previous steps of estimation. There-

fore the necessity of application of the successive approximation method used for the difference solution of multivelocity kinetic equations (2,3) is eliminated. It is also necessary to note that the decribed method may be used for the solution of neutron transport problems in light moderators without siguificant modifications.

Spherically-symmetric problems. In this case the problem is reduced to the solution of the equation (2)

$$\mu \frac{\partial N}{\partial z} + \frac{1-\mu^2}{z} \frac{\partial N}{\partial \mu} + \Sigma N = \mathcal{F} + f \qquad (16)$$

with boundary condition

$$N(a, \mu, \lambda) = \varkappa(\mu, \lambda), \quad \mu < 0. \qquad (17)$$

In the equation (16) the scattering integral $\mathcal{F}$ is determined by the same formula as in the case of the plane-parallel problems (with the difference that z is replaced by τ). Hence, the problem is reduced to the differential equation (16) with the known right side which may be solved by difference methods described in reports (3,4).

Monte Carlo Methods.

Calculation of the flux $\Phi(\vec{z}, \vec{\Omega}, E)$.

It is known that the method of direct modelling of particle motion in a substance is suitable for the comparatively narrow class of problems. A number of modifications of the method was developed for increasing its statistic efficiency.

One of this modifications of the method applicable to the radiation transport problem is a method of local calculation of the flux. In the primary formulation of the method (5,6) it is made use of the fact that the scattered part of the flux $\Phi_s(\vec{z}, \vec{\Omega}, E)$ in a fixed point of space $\vec{z}^* = (x^*, y^*, z^*)$ is equal to the average of raudom variable

$$\begin{cases} \xi = \sum_{n=1}^{\infty} W_n \dfrac{e^{-\tau(\vec{r}_n, \vec{r}^*, E'_n)}}{\rho_n^2} f_s(\vec{r}_n, E_{n-1}, \vec{\Omega}_{n-1}, \vec{\Omega}'_n) \times \Delta(\vec{\Omega}'_n, \vec{\Omega}) \Delta(E'_n, E), \\ \rho = |\vec{r}_n - \vec{r}^*|, \quad \vec{\Omega}'_n = \dfrac{\vec{r}^* - \vec{r}_n}{\rho_n} \end{cases} \qquad (18)$$

where $\vec{r}_n, \vec{\Omega}_n, E_n$ indicate the position of the n-th particle scattering, direction and energy after scattering, respecti-

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vely. $f(\vec{r}_n, E_{n-1}, \vec{\Omega}_{n-1}, \vec{\Omega}'_n)$ is a scattering function, i.e., calculated for the solid angle unity the probability of a particle with the energy before scattering E_{n-1} and direction $\vec{\Omega}_{n-1}$ to scatter in the direction $\vec{\Omega}'_n$ (it possess the energy $E'_n = \varphi(E_{n-1}, \vec{\Omega}_{n-1}, \vec{\Omega}'_n)$). In the formula (18) W_n is a "weight" of the n-th scattering, $\tau(\vec{r}_n, \vec{r}^*, E'_n)$ is an optical distance between $\vec{r}_n$ and $\vec{r}^*$ points for the particles with energy E'_n, $\Delta(x'_n, x)$ is a characteristic function of the interval Δx near. x.

Every individual realization of the process of particle waudering ("history") gives the sampling values of particle coordinates in a phase range $Q(\vec{r}, \vec{\Omega}, E)$ by which may be calculated the corresponding value of random variable ξ. According to the theorem proposed by A.J.Hinchin (7) the convergence on probability of the average from N "Histories" towards the accurate value of the flux at $N \to \infty$ takes place. Due to the fact that the higher moments of the random variable ξ and, in particular, the standard deviation do not exist this convergence is highly slow. The other deficiency of this method is in the necessity of survey of a very great number of histories for the obtaining of a reliable information about angular and energy distribution of the flux.

M.Kalos (8) have proposed the modification of the method mentioned above having the finite standard deviation of the total flux

$$\Phi_s(\vec{r}) = \iint \phi(\vec{r}^*, \vec{\Omega}, E)\, d\vec{\Omega}\, dE$$

This method is based on the inclusion of the multiplier $1/\rho_n^2$, causing the divergence of the standard deviation ξ into the transition probability density of Markov's chain of particle positions in a phase space and, therefore, changes the sampling character.

It may be proposed the modification of Monte Carlo method giving the standard deviation for the total flux and not changing the sampling character. The other valuable

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feature of this method is a possibility of obtaining of energy distribution $\Phi_s(\vec{r}^*, \vec{\Omega}^*, E)$ for the separated fixed direction $\vec{\Omega}^*$.

In the method described above the estimation of the flux is based on its representation in terms of the sum of non-scattered radiation contributions the sources of which are the separated points of particle scattering. It is easy to show that the similar representation takes place if to summarize the contributions of a single-scattered radiation so that

$$\Phi_s(\vec{r}^*, \vec{\Omega}^*, E) = \Phi^{(1)}(\vec{r}^*, \vec{\Omega}^*, E) + M.O. \sum_{n=1}^{\infty} \cdot \xi_n ,$$

$$\xi_n = \frac{W_n e^{-\tau(\vec{r}_n, \vec{r}'_n, E'_n) - \tau(\vec{r}'_n, \vec{r}^*, E)}}{x^2} \Sigma_s(\vec{r}'_n, E'_n) \times$$

$$\times f_s\left(\vec{r}_n, E_{n-1}, \frac{\vec{\Omega}_{n-1} \cdot \vec{x}_n}{x_n}\right) f_s\left(\vec{r}'_n, E'_n, \frac{\vec{\Omega}^* \cdot \vec{x}_n}{x_n}\right) \frac{1}{\frac{dE}{dt_n}}, \qquad (19)$$

where

$$\vec{x}_n = \vec{R}_n - \vec{\Omega}^* t_n, \quad \vec{R}_n = \vec{r}^* - \vec{r}_n, \quad \vec{r}'_n = \vec{r}_n + \vec{x}_n = \vec{r}^*_n - \vec{\Omega}^* t_n ,$$

$$E'_n = \varphi\left(\vec{r}_n, E_{n-1}, \frac{\vec{\Omega}_{n-1} \cdot \vec{x}_n}{x_n}\right),$$

$$E = \varphi\left(\vec{r}'_n, E'_n, \frac{\vec{\Omega}^* \cdot \vec{x}_n}{x_n}\right).$$

In the formula (19) $\Phi^{(1)}(\vec{r}^*, \vec{\Omega}^*, E)$ is a contribution of a single-scattered radiation, M.E. is a symbol of mathematical expectation.

It may be shown that the estimation, following from the formula (19) has the finite deviation for the total flux. The standard deviation of the flux Φ_s does not exist, however, the difficulty connected with a negligible phase volume in a space of directions is eliminated.

As an example, consider the case of the infinite homogeneous medium with isotropic scattering. For the isotropic source the formula (36) leads to the following result:

$$\Phi_s(r) = M.E. \sum_{n=1}^{\infty} \left(\frac{\Sigma_s}{\Sigma_t}\right)^n \mathcal{F}(R_{n-1}),$$

where

$$\mathcal{F}(R) = \frac{2\Sigma_t}{4\pi R} \sum_{K=0}^{\infty} \frac{1}{(2K+1)} E_{2(K+1)}(R\Sigma_t),$$

$$R_n = |\vec{r}_n - \vec{R}_0|, \quad |R_0| = 2, \quad E_j(x) = \int_1^{\infty} \frac{e^{-xt}}{t^j} dt$$

In the table 1 the results of ten sampling each including 200 histories for the case $\Sigma_s/\Sigma_t = 0.75$, $\tau\Sigma_t = 1$, obtained by two methods mentioned above are presented.

The other method of increasing of the efficiency of the local calculation method of the flux is based on that fact that the raudom quantities

$$\xi' = \xi\,\psi_1(\vec{\tau}^*) \qquad \text{and} \qquad \xi'' = \xi\cdot\psi_2(\vec{s}^*)$$

possess the finite dispersion, if $\vec{\tau}^*$ and $\vec{s}^*$ are raudom vectors with non-degenerated distributions in D and S, respectively. Here, ψ_1 and ψ_2 are arbitrary square summable in D functions (on S, respectively), D is a region of a three-dimensional space of the variables x^*, y^*, z^* and S^* is a region of a surface in the same space.

This circumstance allows to calculate the Fourier coefficients of the flux function sampling the orthonormal with weight in D functions as ψ_i function.

If the weight function $P(\vec{\tau}^*)$ is successfully sampled the restoration of the flux function by its Fourier coefficients is performed with a sufficiently high precision. Similar considerations are valid for the flux calculation at the surface S.

The calculations are performed in the following manner. In the beginning of each "history" of a particle raudomly in D it is sampled the position of the detector $\vec{\tau}^*$. The value $\xi(\vec{\tau}^*)$ is calculated. Further, the value $\xi(\vec{\tau}^*)$ may be additionally multiplied by $\psi_1(\vec{\tau}^*)$ and $P(\vec{\tau}^*)$. If $\vec{\tau}^*$ is a vector uniformly distributed in D, then $M.O.\,\xi\cdot(\vec{\tau}^*)f_i(\vec{\tau}^*)P(\vec{\tau}^*)$ is a Fourier coefficient of the flux function, $f_i(\vec{\tau}^*)$ are orthonormal in D with weight $P(\vec{\tau}^*)$ functions ($i=0,1,\ldots,n$).

However, this method of calculation of the Fourier coefficients is not the best one because $M.O.\,\xi\cdot(\vec{\tau}^*)f_i(\vec{\tau}^*)P(\vec{\tau}^*)$ may be a little quantity, while the dispersion $D\{\xi(\vec{\tau}^*)f_i(\vec{\tau}^*)P(\vec{\tau}^*)\}$ may appear significant. In these cases the following generalisation of theorems 1 and 2 of

[9] is useful. Further, we use the notation of this report. Let $f(Q)$ be a square summable with weight $P(Q)(P(Q) \geq 0)$ in D function ($Q \in D$).

Table 1.

Flux No. sampling	(35)	(36)
1	0.0678	0.0656
2	0.0689	0.0660
3	0.0624	0.0634
4	0.0843	0.0721
5	0.0596	0.0637
6	0.0688	0.0628
7	0.0842	0.0689
8	0.0746	0.0660
9	0.0896	0.0761
10	0.0567	0.0615
Average	0.0709	0.0666
Exact value	0.0665	

Consider a generalized interpolating polynomial $P(Q)=C_0 f_0(Q)+C_1 f_1(Q)+C_n f_n(Q)$, constants C_i $(i=0,1,\ldots,n)$ are defined from the conditions

$$P(Q_j) = f(Q_j),$$

where Q_j are certain fixed points of a region D. The following theorem is valid.

Theorem 1. If the points Q are randomly sampled in D with the density of a probability $W_n(Q_0, Q_1, \ldots, Q_n)$, $W_n = \frac{1}{(n+1)!}\left[\det \| f_0(Q_m)\sqrt{P(Q_m)}, \ldots, f_n(Q_m)\sqrt{P(Q_m)} \|_0^n \right]^2$, then

$$M.O.C_i = \int_D P(Q) f(Q) f_i(Q)\, dQ, \quad i=0,1,\ldots,n.$$

Under the same assumptions it is valid.

Theorem 2. A standard deviation of the random value C_i is equal to

$$\int_D P(Q)\left[f(Q) - \sum_{i=0}^{n} M.O.C_i\, f_i(Q)\right]^2 dQ.$$

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The results of the calculation of a numerical gamma spectrum in the air by the local calculation method are presented in the Table 2.

The similar values obtained by a linear algebraic interpolation by random points are presented in the Table 3. In a given case the weight function is not introduced.

The four groups of data are presented in these tables. Each group is obtained as a result of 1,000 tests.

A point isotropic monoenergetic source with E_o=1.25Mev is used as a gamma-ray source. The source-detector distance is 30 meters. 0-1.25Mev energy interval is divided into eight uniform groups. If the quantity presented in the tables 2 and 3 multiplied by 10^{-9}, then we obtain a number of gamma-rays /second/cm^2, which is belonged to the given energy group.

Table 2.

No. of sampling / No. of group	1	2	3	4
1	1.580	0.925	0.737	0.808
2	1.039	1.162	1.123	0.975
3	0.450	0.296	0.342	0.496
4	0.826	0.303	0.104	0.228
5	0.123	0.105	0.214	0.149
6	0.138	0.594	0.376	0.129
7	0.053	0.145	0.110	0.112
8	0.073	0.208	0.164	0.022
Integrated flux	4.832	3.322	4.107	2.917

Table 3.

1	1.006	0.864	0.678	0.827
2	1.463	1.359	0.910	1.322
3	0.335	0.437	0.753	0.173
4	0.301	0.127	0.117	0.292
5	0.461	0.415	0.289	0.576
6	0.398	0.095	0.281	0.356
7	0.192	0.050	0.694	0.000
8	0.000	0.417	0.087	0.460
Integrated flux	3.986	3.763	3.809	4.006

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Calculation of the functionals from the flux $\Phi(\vec{r},\vec{\Omega},E)$.

1. Monte Carlo method is often used for calculation of the functionals f from the flux $\Phi(\vec{r},\vec{\Omega},E)$ in a space-energy region in the inhomogeneous problems of a neutron transport theory,

$$f=\int d\vec{\Omega}_0 \int_{\Delta E_0} dE_0 \int_{\Delta V_0} d\vec{r}_0\, S(\vec{r}_0,\vec{\Omega}_0,E_0)\int d\vec{\Omega}\int_{\Delta E} dE\int_{\Delta V} d\vec{r}\, f(\vec{r},\vec{\Omega},E)\,\phi(\vec{r}_0,E_0,\vec{\Omega}_0;\vec{r},\vec{\Omega},E) \qquad (20)$$

where $\phi(\vec{r},\vec{\Omega},E;\vec{r}_0,\vec{\Omega}_0,E_0)$ is a Green function of the corresponding transport equation. As a rule the regions of determination of the source $S(\vec{r}_0,\vec{\Omega},E_0)$ and the function $f(\vec{r},\vec{\Omega},E)$ are greatly differed, therefore, the increase of a statistic efficiency is achieved when a sampling of the original coordinates of history is performed in accordance with a adjoint function in relation to functional f [10]. Accurate finding of a adjoint function is nor less complex problem than the original one, therefore for evaluation of the functional f it is naturally to try to construct such adjoint function. If this function $\tilde{Q}(\vec{r}_0,\vec{\Omega}_0,E_0)$ is constructed, the functional f may be written in a following manner [1]

$$f=\int d\vec{\Omega}_0\int_{\Delta E} dE_0\int d\vec{r}_0\,\tilde{S}(\vec{\Omega}_0,\vec{r}_0,E_0)\,R(\vec{r}_0,\vec{\Omega}_0,E_0)\times$$
$$\times\int d\vec{\Omega}\int dE\int d\vec{r}\, f(\vec{r},\vec{\Omega},E)\,\phi(\vec{\Omega},\vec{r},E;\vec{\Omega}_0,\vec{r}_0,E_0)], \qquad (21)$$

where

$$\tilde{S}(\vec{r}_0,\vec{\Omega}_0,E_0)=\frac{S(\vec{r}_0,\vec{\Omega}_0,E_0)\,\tilde{Q}(\vec{r}_0,\vec{\Omega}_0,E_0)}{\int d\vec{\Omega}_0\int dE_0\int d\vec{r}_0\, S(\vec{r}_0,\vec{\Omega}_0,E_0)\,\tilde{Q}(\vec{r}_0,\vec{\Omega}_0,E_0)};$$

$$R(\vec{r}_0,\vec{\Omega}_0,E_0)=\frac{\int d\vec{\Omega}\int dE\int d\vec{r}\; S\cdot Q}{Q(\vec{r}_0,\vec{\Omega}_0,E_0)}$$

and sampling of original coordinates of the history may be performed in accordance with density S and initial weight R.

When determining the parameters of the resolved neutron levels according to the results of the radiation capture measurements the necessity is appeared to calculate the functionals of a type (20) which represent in this case the average absorption rate in a planar sample of thickness H of the neutrons from a plane source with a given spectrum

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$S(E_0)$ in the energy interval ΔE_0 entirely comprising the resonance considered (11,12,13). In this problem Monte Carlo method is used for evaluation of the average absorption rate of neutrons suffering scattering P_s which has the following form:

$$P_s = \int_{\Delta E_0} dE_0\, S(E_0) \int_{-1}^{+1} d\mu \int_{\Delta E} dE \int_0^H dz\, \Sigma_j(E)\, \phi_s(z,\mu,E,E_0) \qquad (22)$$

where $\phi_s(z,\mu,E,E_0)$ is a neutron flux after a single scattering. The application of a adjoint function for estimation of a functional (22) is of importance as a region of determination of the source ΔE_0 always considerably exceeds the region where a adjoint function $Q(E_0) = \int_{-1}^{+1} d\mu \int_{\Delta E} dE \int_0^H dz\, \Sigma_j(E)\, \phi(z,\mu,E,E_0)$ is differed from zero.

For a construction of the approximate function $\tilde{Q}(E_0)$ take a following model of the neutron transport. Let us believe that the neutrons diffuse-through a sample of an effective thickness H^* without a collision but at the boundary some fraction of neutrons is reflected with an energy change by the magnitude of an average logarithmic loss ξ.

Then an approximate adjoint function is written down in the following manner:

$$\tilde{Q}(E_0) = \sum_{k=0}^{\infty} P_{k+1} P^k \prod_{\ell=0}^{n} S_\ell, \qquad (23)$$

where

$$P_K = \frac{\sigma_s(E_K)}{\sigma_t(E_K)} \left(1 - e^{-H^* \Sigma_t(\Sigma_K)}\right); \quad S_K = \frac{\sigma_s(E_K)}{\sigma_t(E_K)} \left(1 - e^{-H\Sigma(E_K)}\right).$$

$$E_K = E_0\, e^{-K\xi}$$

The normalized adjoint function is always used in an importance sampling of original coordinates, therefore the deviations of parameter values H^* and P from H and I, respectively do not influence greatly on the efficiency of approximate function application.

The main feature of this function - a maximum displacement relative to a resonance energy - is well interpreted by introduction of an average logarithmic loss ξ into the expression (23). In the case of realization of Monte Carlo method for estimation of the functionals of a type (22) it

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is important that introduction of importance sampling by a adjoint function (23) does not prevent from the application of the similar trajectory method for the simultaneous correlated calculation of a wide range of sample thicknesses.

This method of improvement of a statistic efficiency is highly useful in a problem of calculation of a resonance absorption in a stratified activation detectors [14]. With the help of one approximate adjoint function for a sample of intermediate thickness it is managed to carry on the estimation of activation absorption at a single level in the inner detector layer simultaneously for a wide range of filter thicknesses. These calculations were carried out for some activated isotopes with the purpose of obtaining some optimal sample thicknesses for the neutron spectrum measurements within a reactor.

2. The other way of statistic efficiency improvement of Monte Carlo method in the case of estimation of functionals (20) when its region of determination $\Delta E \times \Delta V$ is considerably less than the region of determination of the flux density $\varphi(\vec{r},\vec{\Omega},E)$ is based on obtaining of some information about an importance function in the process of sampling and its application for choice of further fate of a neutron. This is a method of splitting.

If in the process of random sampling before next collision a neutron has the coordinates $(\vec{r}_n,\vec{\Omega}_n,E_n)$ not belonging to the region of a functional determination, then the probability to contribute to this functional after collision is equal to corresponding part G_f of a single volume of hypercube. Sampling randomly and uniformly the point ρ from this part of a hypercube we obtain at every collision a contribution to functional f and one of the branches for a further neutron fate. The second branch ensuring the compensation of a bias of such sampling corresponds to the random point from the hypercube region additional to G_f. Sampling between these two branches is performed in a random manner in accordance with its weight factors G_f and

$1-G_1$, respectively.

The application of the method of splitting is highly useful in the case of Monte Carlo calculation of the influence function for monoenergetic isotropic source located at the block surface in the infinite cylindrical lattice. This function being analogous to the Plachek's function in a homogeneous medium is interest in the resonance absorption problems in heterogeneous media.

In this problem the following quantity was considered as a functional f.

$$f = \int d\vec{\Omega} \int_{V\delta\Lambda} d\vec{\tau} \int dE \, \Sigma_t(\vec{\tau}, E) \, \phi(\vec{\tau}, \vec{\Omega}, E).$$

representing the average density of collisions in a block in the energy interval $E_i \leq E \leq E_{i+1}$

As a rule the determination of boundaries of a multidimensional region G_f and sampling of random point from this region is a very difficult task, therefore, it is desirable to arrange the splitting by one of the variables.

In the case considered a region ensuring the contribution to the functional f is defined by the following condition:

$$| \sin \varphi_{n+1} | \leq \frac{R_{\delta\Lambda}}{|\tau|} \equiv A,$$

where φ_{n+1} is a azimuthal angle of the neutron velocity direction after scattering; $R_{\delta\Lambda}$ is a block radius. The splitting is performed by one variable, μ, a cosine of scattering angle in a system of inertia. There, one can obtain the following ratios for weights of the branches.

$$G_1 = \frac{1}{2} | \mu_0^{(1)} - \mu_0^{(2)} |, \quad G_2 = 1 - G_1,$$

where $\mu_0^{(1,2)} = \dfrac{\eta_{1,2}\sqrt{(M^2+1)\, x_{1,2}^2 \sin^2\Theta + M^2 \eta_{1,2}^2 - x_{1,2}^2 \sin^2\Theta}}{M(\sin^2\Theta\, x_{1,2}^2 + \eta_{1,2}^2)}$

while $\eta_{1,2} = A\xi' \pm \sqrt{1+A^2} \cdot \eta'$; $x_{1,2} = A\cos\varphi \pm \sqrt{1-A^2}\sin\varphi$,

$\xi' = \sin\alpha \sin\varphi_n + \cos\alpha \cos\varphi_n \cos\Omega_n$; $\eta' = \sin\alpha \cos\varphi_n - \cos\alpha \cdot \sin\varphi_n \cos\Omega_n$

M– is a nuclear mass.

Calculation of a heterogeneous Plachek's function showed a considerable improvement of a statistic efficiency of Monte Carlo method at the expence of application of the split-

ting techniques particular in a case of very thin blocks.

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